Safety distances in airsoft with proper consideration of atmospheric drag

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ABSTRACT

In this paper the relationship between safety distances, projectile mass and muzzle velocities in airsoft is reviewed. The physical background is studied, especially the Reynolds number, the drag coefficient and the quadratic drag model of air resistance. A physical model is presented and a safety table is proposed based on that physical model, with the addition of a cap on muzzle energy due to safety and legal concerns.

Introduction

In airsoft, we want all shots impact energy to not exceed a safety threshold to ensure the health and safety of all game participants. To allow for a greater range of the airsoft weapons the energy of the projectile is not necessarily counted by the muzzle, but can instead be counted at a farther distance where atmospheric drag has brought down the energy of the projectile to safe levels. The shooter is not allowed to fire at anything within this "safety distance" to ensure that the impact energy of his shots are always below an energy level deemed safe. The energy of the shots are routinely measured before any airsoft game.

While one could imagine the projectiles energy being measured at the expected safety distance, which would simplify the calculations to triviality by not having to deal with modeling the atmospheric drag, it is not practical to do with normal measuring equipment at games. Instead the projectile energy is measured directly at the muzzle of the weapon and then referenced against a precalculated table with various values of projectile weights, impact energy safety thresholds, safety distances and muzzle velocities. The objective of this paper is to produce such a table grounded in a proper understanding of the underlying physics.

Fluid mechanics is hard. There is a reason why companies still use wind tunnels rather than computation and simulation only and for our purposes in airsoft we will always have to resort to simplifications and estimations unless we make the effort to do real measurements at the safety distance, eliminating the need to model atmospheric drag. That said, the model proposed in this paper is probably good enough for our purposes. Common assumptions and estimations in fluid mechanics include linear drag for slow speeds with laminar flow and quadratic drag for higher speeds with turbulent flow. Computing the Reynolds number can help determine which flow regime a problem resides in. The Reynolds number for airsoft shots is far outside the regime of laminar flow, indicating that the problem is turbulent and that the quadratic drag model should be used.

The drag coefficient is an important factor that incorporates most of the complexities of fluid mechanics for the drag problem, including the shape of the object. It depends only on the Reynolds number with the relationship, determined empirically, illustrated in figure 1. Note that airsoft shots are found firmly within the constant region, which supports our use of the quadratic drag model and allows us to assume a constant drag coefficient value for all relevant parameter sets found in airsoft.

If we where interested in computing the ballistic arc of the projectile, considering both the drag force and the gravitational force, we would not be able to find an analytic solution to the equations of motion and would have to fall back on a numerical solution. However, we are not interested in computing the arc but rather the projectile energy as a function of distance traveled and for that energy the contribution of the gravitational force is small enough to be neglected. This is great, because it simplifies the equations and allows us to find a closed, analytical solution.

One factor that is ignored in this paper is the rotation of the airsoft projectiles. Most airsoft weapons use a device called a "hop-up", giving the projectiles a backspin. This spin takes advantage of another fluid mechanical process called the "Magnus effect", producing an upward force balancing the gravitational pull and extending the airsoft weapons range considerably. This spin obviously affects the aerodynamic properties of the projectile, but in this paper it is assumed that it does not have a significant impact on the breaking force of the atmospheric drag and therefore does not contribute considerably to the impact energy of the projectile.

Results

The important expressions derived in this paper can be summarized as

$$u_0 = \sqrt{\frac{2E_1}{m}} e^{C\frac{x_1}{m}} \qquad x_1 = \frac{m}{C} \ln\left(u_0 \sqrt{\frac{m}{2E_1}}\right) \qquad C = \frac{1}{2} C_D \rho A$$

with the factor explanations and values found in table 1.

Taking a typical speed of an airsoft projectile to be 100 m/s the Reynolds number of a typical airsoft shot is $Re \approx 4 \cdot 10^4$. This puts it in the center of the constant region of the drag coefficient, as can be seen in figure 1.

The safety tables was computed with the code in listing 1 and can be seen as tables 2 and 3. The values in table 2 are the true muzzle velocity values that gives the proper impact energy at the specified distances, while table 3 caps the muzzle energy at more reasonable safety thresholds. Note that under Swedish weapons law non-license automatic weapons are not allowed to exceed a muzzle energy of 3 J and non-license non-automatic weapons may not exceed 10 J.

Property	Symbol	Value	Unit	Comment
Air density[1]	ρ	1.225	kg/m^3	15°C, 1 atm
Air dynamic viscosity[1]	μ	$1.802 \cdot 10^{-5}$	N s/m^2	15°C, 1 atm
Characteristic length	L	0.006	m	Diameter of the projectile
Area	A	$1.131 \cdot 10^{-4}$	m^2	Cross section of the projectile
Drag coefficient[2]	C_D	0.4	unitless	See figure 1.
Kinetic energy safety threshold at impact	E_1	1.2	J	Variable, here always set to 1.2 J.
Kinetic energy safety threshold at muzzle	-	$1.2, 1.45, 1.7, 2.2, 3, 4$	J	Variable, constant per x_1
Distance at which the projectile reaches the energy E_1	x_1	-	m	Variable
Projectile mass	m	-	kg	Variable

Table 1. The variables and constant values used in the computations.

Class	Safety distance	.20	.25	.28	.30	.34	.36	.40	.43	.45	.48	.50
CQB	0	109.5	98.0	92.6	89.4	84.0	81.6	77.5	74.7	73.0	70.7	69.3
Assault 1	5	130.3	112.5	104.8	100.4	93.0	89.9	84.5	81.0	78.9	76.0	74.3
Assault 2	10	154.9	129.3	118.6	112.7	103.0	99.0	92.1	87.8	85.2	81.7	79.6
DMR	20	219.0	170.5	151.9	141.9	126.3	120.0	109.5	103.1	99.4	94.4	91.4
Sniper 1	30	309.6	225.0	194.5	178.8	154.8	145.4	130.2	121.1	115.9	109.0	105.0
Sniper 2	40	437.8	296.8	249.1	225.3	189.8	176.3	154.9	142.3	135.2	125.9	120.6

Table 2. The safety table, with no max muzzle energy. Note that the muzzle velocity values in the lower left corner becomes unreasonably high.

Class	Safety distance	.20	.25	.28	.30	.34	.36	.40	.43	.45	.48	.50
CQB	0	109.5	98.0	92.6	89.4	84.0	81.6	77.5	74.7	73.0	70.7	69.3
Assault 1	5	120.4	107.7	101.8	98.3	92.4	89.8	84.5	81.0	78.9	76.0	74.3
Assault 2	10	130.4	116.6	110.2	106.5	100.0	97.2	92.1	87.8	85.2	81.7	79.6
DMR	20	148.3	132.7	125.4	121.1	113.8	110.6	104.9	101.2	98.9	94.4	91.4
Sniper 1	30	173.2	154.9	146.4	141.4	132.8	129.1	122.5	118.1	115.5	109.0	105.0
Sniper 2	40	200.0	178.9	169.0	163.3	153.4	149.1	141.4	136.4	133.3	125.9	120.6

Table 3. The safety table, with caps set on muzzle energy. To own an automatic weapon with a muzzle energy above 3 J in Sweden one needs a license.

Discussion

In this paper the physical background of the airsoft safety table has been reviewed. It is interesting to note how much impact the different factors have on the allowed muzzle velocity, where we can note a very strong dependence on the projectile mass, a strong dependence on the safety distance and a weak dependence on the impact energy. The exponential nature of the energy decay over distance lead to that when both mass and distance simultaneously promote a high muzzle velocity it gets unreasonably high, making it necessary to introduce a cap for both safety and legal reasons.

It would be interesting in future work to test the model presented in this paper empirically. Using a chronograph with a large measuring cross section it would be possible to measure the projectile velocity directly at the safety distance. Measuring both this value and the muzzle velocity with all other parameters constant would provide excellent reference data for a comparison with the theoretical data computed in this paper.

Hopefully this review has clarified some of the confusion that has previously existed around the safety table and the author hope that this review will contribute to safety, fairness and joy in the airsoft community.

Methods

Reynolds number is an important property of any fluid mechanical problem

$$Re = \frac{\rho L u}{\mu}$$

and can be computed from the density ρ, the characteristic length L (the diameter for a sphere), the flow velocity u and the dynamic viscosity μ. It gives a ratio between inertial forces and viscous forces in a fluid flow and can help determine if it is laminar, transient or turbulent.

- Laminar: $Re < 2400$
- Transient: $2400 < Re < 4000$
- Turbulent: $Re > 4000$

An important result in fluid dynamics is that the drag coefficient is a function only of the Reynolds number of the fluid flow about the object. That is,

$$C_D = C_D(Re)$$

This functional relationship has no closed form. However, the relationship has been established numerically based on experimental data. See figure 1 for a schematic diagram of the drag coefficient's dependence on the Reynolds number. Airsoft shots typically have a Reynolds number of about $4 \cdot 10^4$, placing them in the constant region of the drag coefficient and validating our assumption of a quadratic drag model.

$$F_D = \frac{1}{2} C_D \rho A u^2 = C u^2 \qquad F = m u'(t) = -F_D = -C u^2(t)$$

We substitute $C = m\lambda$ for now in order to clean up the calculations and arrive at the fundamental differential equation defining our problem.

$$u'(t) = -\lambda u^2(t)$$

We note that this differential equation is separable and thus relatively easy to solve.

$$-\int \frac{du}{\lambda u^2} = \int dt = t + c = \frac{1}{\lambda u} \qquad u(t) = \lambda^{-1}(t+c)^{-1} \qquad u(0) = (\lambda c)^{-1} = u_0 \implies c = (\lambda u_0)^{-1}$$

$$u(t) = (\lambda t + u_0^{-1})^{-1}$$

Having expressed the velocity as a function of time we now want to use it to express the kinetic energy as a function of time.

$$E = \frac{mu^2}{2} \qquad E(t) = \frac{m}{2}(\lambda t + u_0^{-1})^{-2}$$

We note here that the energy decrease with time, as expected. We now want to calculate the time t_1 when the energy has reached the level E_1.

$$E(t_1) = E_1 \qquad t_1 = \lambda^{-1}\left(\sqrt{\frac{m}{2E_1}} - u_0^{-1}\right)$$

Let us now instead consider the distance traveled x. Knowing the velocity as a function of time $u(t)$ we can integrate and get the distance as a function of time $x(\tau)$ as well.

$$x(\tau) = \int_0^\tau u(t)dt = \int_0^\tau \frac{dt}{\lambda t + u_0^{-1}} = \left[\lambda^{-1}\ln(\lambda t + u_0^{-1})\right]_0^\tau = \lambda^{-1}\left(\ln(\lambda\tau + u_0^{-1}) - \ln(u_0^{-1})\right) = \lambda^{-1}\ln(\lambda u_0 \tau + 1)$$

Now we can use the time t_1 that we calculated earlier to calculate how far the projectile has traveled when it reaches the energy level E_1.

$$x_1 = x(t_1) = \lambda^{-1}\ln\left(\lambda u_0\left(\lambda^{-1}\left(\sqrt{\frac{m}{2E_1}} - u_0^{-1}\right)\right) + 1\right) = \lambda^{-1}\ln\left(u_0\sqrt{\frac{m}{2E_1}}\right)$$

In the safety table we want to fix the values of impact energy E_1, safety distance x_1 and mass m to get the initial velocity u_0, so lets solve for u_0.

$$\lambda x_1 = \ln\left(u_0\sqrt{\frac{m}{2E_1}}\right) \qquad e^{\lambda x_1} = \left(u_0\sqrt{\frac{m}{2E_1}}\right) \qquad u_0 = \sqrt{\frac{2E_1}{m}}e^{\lambda x_1}$$

Finally we arrive at our final expression by re-substituting $C = m\lambda$.

$$u_0 = \sqrt{\frac{2E_1}{m}}e^{C\frac{x_1}{m}}$$

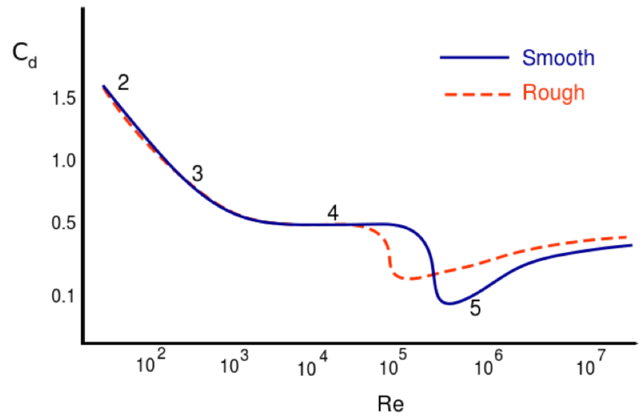

Figure 1. The drag coefficient C_D as a function of the Reynolds number Re. The relationship is determined empirically. Note that the Reynolds number for an airsoft shot is typically around $4 \cdot 10^4$, placing it firmly in the constant region of the drag coefficient. Image credit: NASA[2]

```python
import numpy as np

# ********** Subjective inputs **************************************************
masses = np.array([20, 25, 28, 30, 34, 36, 40, 43, 45, 48, 50])/100000 # kg
distances = np.array([0, 5, 10, 20, 30, 40]) # m
energiesMuzzle = np.array([1.2, 1.45, 1.7, 2.2, 3, 4]) # J (max energy at muzzle)
energyImpact = 1.2 # J (max energy at safety distance)

if distances.size != energiesMuzzle.size:
    print("Number of distances and muzzle energy does not match!")

# ********** Output ***********************************************************
muzzleVelocities = np.array(np.ones([distances.size, masses.size])); # m/s

# ********** Parameters *******************************************************
density = 1.225 # kg/m^3 (15 degC, 1 atm)
dynamicViscosity = 1.802*10**-5 # N s/m^2 (15 degC, 1 atm)
characteristicLength = 0.006 # m (diameter of projectile)
area = np.pi * (characteristicLength/2)**2 # m^2
dragCoefficient = 0.4 # unitless

constant = 0.5 * dragCoefficient * density * area

# ********** Energy computation functions *************************************
def velocityFromEnergyWithDrag(m, x, E, k):
    return np.sqrt(2*E/m)*np.exp(k*x/m)

def velocityFromEnergy(m, E):
    return np.sqrt(2*E/m)

def velocityMax(mass, distance, Eimpact, Emax, const):
    vFromDrag = velocityFromEnergyWithDrag(mass, distance, Eimpact, const)
    vMuzzleMax = velocityFromEnergy(mass, Emax)
    return np.min([vFromDrag, vMuzzleMax])

# ********** Population of the table ******************************************
for i_class in range(distances.size):
    for i_mass in range(masses.size):
        mass = masses[i_mass]
        distance = distances[i_class]
        energyMuzzle = energiesMuzzle[i_class]
        muzzleVelocities[i_class, i_mass] = \
            velocityMax(mass, distance, energyImpact, energyMuzzle, constant)

# ********** Print output *****************************************************
np.set_printoptions(linewidth=100)
print(np.array2string(muzzleVelocities, precision=1))
```

Listing 1. The python code used to compute the table.

References

1. EngineersEdge. Viscosity of air, dynamic and kinematic. *Engineers Edge* https://www.engineersedge.com/physics/viscosity_of_air_dynamic_and_kinematic_14483.htm (2019).
2. Hall, N. Drag of a sphere. *NASA* https://www.grc.nasa.gov/WWW/k-12/airplane/dragsphere.html (2015).